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STAT

Attached is the document that I spoke with about today. As we discussed I am sending it to you so you might anticipate some of the questions during our visit on 24 October. Certainly this document should not be interpreted as evidence of our dissatisfaction of the quality of your performance under this effort, but would hopefully be considered as an aid, derived from the experiences that we have had in the past.

If there are any questions concerning the paper please do not hesitate to contact me.

STAT

4 October 1966

Comments on "Improved Screen for Rear Projection Viewer"

I have read the Corning report on Phase II (Theoretical Studies), and consider it thorough, well-prepared, and clearly indicative of the technical capabilities of the personnel who participated in the study. They have examined the problem and established the current limits of technology. The base now exists for the much-needed research to produce the screens required. I take no exception to their general results or quarrel with their method.

However, I do have reservations about their proposed method of MTF determination. I have no reservation about the necessity for transfer function determinations, but my past experience with their proposed technique indicates the high probability of major uncertainties. I have attached the analysis I performed of this technique in late 1962, which lists many of the difficulties in the engineering implementation.

I believe the proposed technique to be fundamentally imprecise. At no time is there a given frequency being displayed or analyzed -- only something which is describable as an instantaneous spatial frequency. Thus the measuring S/N is extremely poor at all frequencies. I further do not agree with their assertion that while available, simple sine-wave patterns cannot be used. Measurement of MTF using only a half-dozen sine-wave targets will give a curve which will be inherently more accurate than the method proposed -- unless Corning knows something that I do not (a fact eminently possible, and inasmuch as my knowledge dates back to 1962 in the matter, highly probable).

There are two considerations which do not appear as if they had been established soundly:

1) If Corning really wants to test the screens in the manner in which they will be used, they should not demagnify the sinusoidal patterns: this places the image in convergent light, whereas the images are always viewed (at least in projection) in divergent illumination. I would expect the scattering and interference effects to be vastly different. The patterns when magnified will require much higher spatial frequencies than they currently are considering, and the problem of producing the frequency-modulated patterns is far more difficult than indicated. As a matter of fact, the necessity for polarizer rotation or film translation motion to be accelerated is not clearly stated (and this is necessary to frequency-modulate the patterns they propose).

2) Because the test screens will vary in physical properties, the scattering will change with each one. Thus the problem of Callier-Q

3. MODULATION TRANSFER FUNCTIONS AND THRESHOLD CHARACTERISTICS OF EMULSIONS

Although serious investigation is being made of the modulation transfer functions of emulsions, the extreme difficulty of isolating the effects of the emulsion's D-log E characteristic, neighborhood effects, film grain, spread function, and the response of the human eye strongly suggest the desirability of an alternate approach — determination of the emulsion's threshold characteristic — which accounts for the complex interaction of all these effects. Accordingly, the work now in progress embraces both areas.

3.1 MODULATION TRANSFER FUNCTION

During this report period, study has been made of a moving-film technique for imaging one-dimensional, sine-wave distributions on emulsion surfaces. This work has led to the conclusion that spatial frequencies up to 300 or 400 lines per millimeter can be imaged for numbers of lines in excess of 10 but probably not more than 100.

Further study has been made of the previously reported, coherent light method for determining the modulation transfer of emulsions. This work has shown that the gamma for Pan-X and Tri-X film is somewhat higher than that previously assumed, because the measured density must be characterized as specular rather than diffuse. These two areas of inquiry are discussed in the following sections.

3.1.1 Moving-Film Technique

A method (originally reported by Ingelstam¹) for producing one-dimensional, sine-wave density distributions has been studied, and the technique is summarized in this report. Briefly, light from a slit is sinusoidally time modulated by a rotating polarizer-analyzer system, imaged at a reduced size on film, and converted to spatial variance by moving the film past the slit image with constant velocity. The contrast and frequency can be varied easily and the light level adjusted for correct image exposure on the requisite portion of the emulsion's characteristic curve, or for exposure equalization at all frequencies.

A schematic outline of the basic system is shown in Fig. 3-1. L1 and L2 are lenses which cause the monochromatic illumination to be focused at the slit. Polarizer P₂ is rotated at a constant angular velocity, or, alternatively, P₁ and P₂ can be counterrotated with respect to one another at half the velocity used for P₂. The intensity variation (aerial image contrast) of the slit image can be controlled by the quarter-wave plate. If this plate is not used, the polarizing efficiencies of the elements themselves determine the contrast. There is no effective dynamic control on contrast when heterochromatic illumination is to be used since the quarter-wave plate suffices for single wavelength only.

The normalized time-varying intensity distribution, I, of the slit image is a function of the angle, θ , between the polarization directions of P₁ and P₂, and is written

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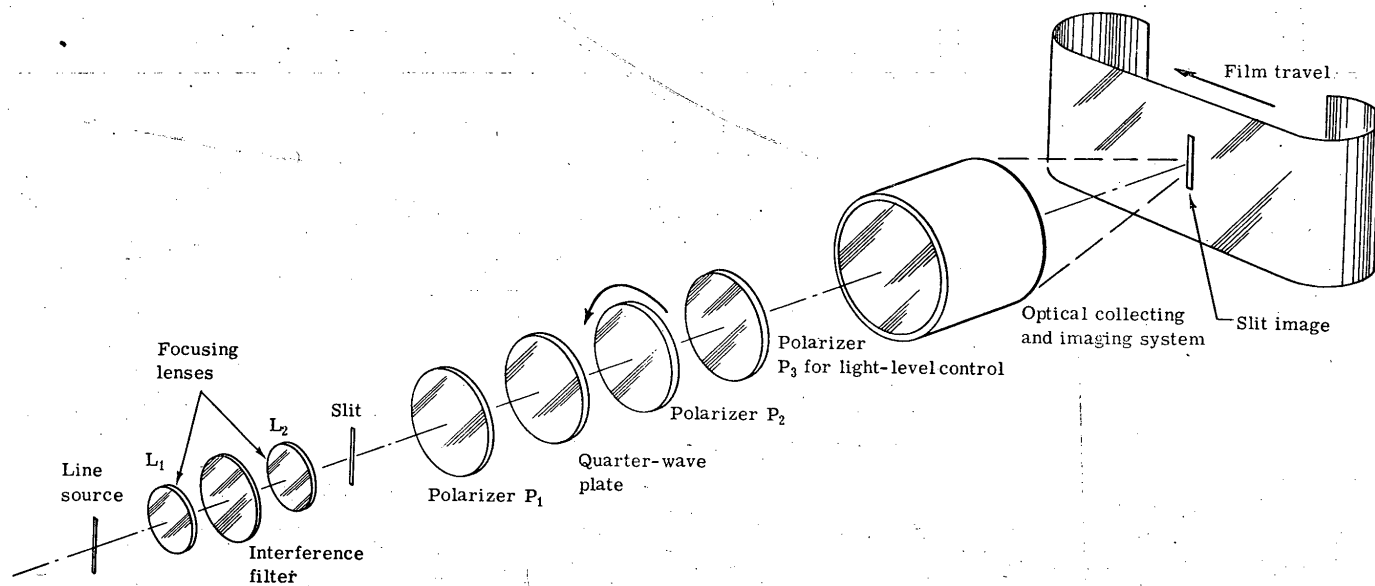


Fig. 3-1 — Optical schematic of system for generating a time-varying sinusoidal intensity distribution and converting this distribution to spatial variance on film

$$I(\theta) = 1 + M \cos (2\theta) \quad (3.1)$$

Modulation, M , in the absence of quarter-wave plate control is a function of the polarizer combination

$$M = \frac{H_{\max} - H_{\min}}{H_{\max} + H_{\min}} \quad (3.2)$$

where H_{\max} = transmittance when polarizing directions are parallel
 H_{\min} = transmittance when polarizing directions are perpendicular

If P_2 is rotated relative to P_1 at a constant angular velocity, the angle θ varies with time according to the velocity relation

$$\theta = 2\pi ft \quad (3.3)$$

where f = rotational speed in revolutions per unit time

Combining Eqs. 3.1 and 3.3

$$I(t) = 1 + M \cos (4\pi ft) \quad (3.4)$$

This equation describes the intensity of the time-varying slit image at the focal plane of the collecting and imaging optical system. A photographic emulsion in this plane is moved in a direction perpendicular to the long dimension of the slit. The spatial variance in the direction of motion is described by the expression

$$x = vt \quad (3.5)$$

The film velocity, v , is assumed to start at the origin. The latent image becomes a spatially varying sinusoid whose mathematical characterization can be obtained by combining Eqs. 3.4 and 3.5.

$$I(x) = 1 + M \cos \left(4\pi \frac{f}{v} x \right) \quad (3.6)$$

This expression can be written in terms of spatial frequency, k , as

$$I(x) = 1 + M \cos (2\pi kx) \quad (3.7)$$

where $k = 2(f/v)$ in cycles per unit length.

The modulation of the aerial image, M , must be replaced, since it is modified by the modulation transfer characteristics of the photographic material. Thus, M in Eqs. 3.6 and 3.7 represents the modulation of the filmed, photographic sine wave.

Fig. 3-2 shows the relationship among k , f , and v in terms of values having practical significance. As an example, consider the requirement of producing a sinusoidal image of 100 cycles per millimeter. If a pattern 1/10 inch wide is suitable, the polarizer may be rotated at a speed of 125 rpm, and the film moved at a speed of 1/10 inch per minute for 1 minute. These relationships can be derived from the plot of Fig. 3-2. The same 1/10-inch-wide pattern can be achieved

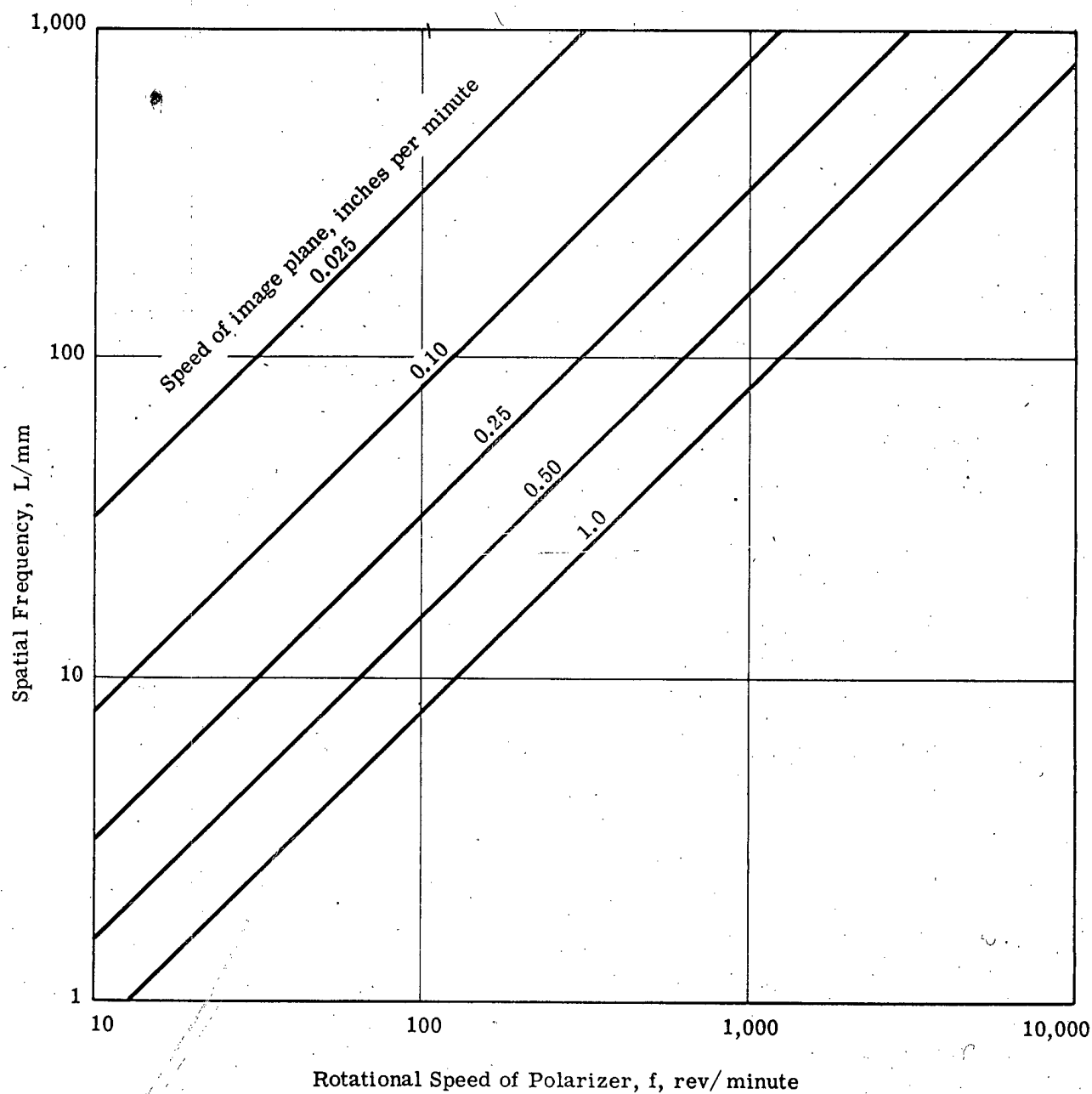


Fig. 3-2 — Parametric variation of spatial frequency with (1) polarizer rotational speed, and (2) image or film plane translational speed

at 1,250 rpm, with a film velocity of 1-inch per minute for 6 seconds. There is no indication at this time as to which choice is better, since the precision and stability of the necessary rotating and translating mechanisms are unknown.

Polarizer P_3 , an adjustment for light level, is required as a means of adjusting the exposure to the film speed and to the sensitometric characteristics. Since exposure decreases with increasing frequency, such an adjustment is also necessary when exposures are equalized as a function of frequency. This relationship may be seen from a comparison of the relative exposures of, for instance, 10 and 100 cycles per millimeter. At the same light level, 1 cycle of the 100-cycle-per-millimeter pattern is exposed to the film in 1/10 the time required for 1 cycle of the 10-cycle-per-millimeter pattern. The film at 100 cycles per millimeter thus has a photographic exposure only 1/10 that received for the 10-cycle-per-millimeter pattern. By attenuation of the stronger light level, the exposures may be equalized. Since the motion of P_3 is angular, the use of a vernier control permits an equalization adjustment that is quite precise and reproducible.

Since the optics pass a time varying signal, one obvious advantage of this technique is that their contrast modifying characteristics need not be known. Furthermore, even though the optics have the same frequency response for all spatial frequencies appearing in the filmed image, it is important — to prevent degradation of the resultant sinusoidal pattern — that the lens used to form the reduced slit image on the film be of sufficient quality to reduce flare and skirts at the focal plane. The major effect of any contrast modifying characteristics is the reduction of input contrast. However, this effect could be determined and need not be a decisive factor in ascertaining system limits.

The major disadvantages of this system are those inherent in the rotating and translating mechanisms. Vibration, gear chatter, gear backlash, etc., all become pronounced at high spatial frequencies. The capability of maintaining constant angular motion despite voltage changes is also a determining factor in the design. Although the magnitudes of these effects are not known at this time, they could be minimized through careful attention during the design stage.

From an engineering and fabrication point of view, the feasibility of this technique is demonstrated by the equipment presently in operation.^{1,2} The number of cycles which can be accurately laid down on the film at a given spatial frequency are dependent on the mechanical parameters. The accuracy to which the contrast of the aerial image can be known is a function of the imaging optics and the ability to minimize the spread of the reduced image. The use of microdensitometer optics will help alleviate this problem, but the precise limits are not presently known. It is estimated that this technique could be used for spatial frequencies up to 300 or 400 cycles per millimeter for numbers of cycles in excess of 10 but probably not more than 100.

A variation of the above method has been noted recently by Shaw.² His technique accelerates (or decelerates) the movement of the rotational or translational functions. The resultant density distribution on the film is a continuous analog to the Fresnel zone plate, in that the waves close in as the pattern size increases. Furthermore, the pattern possesses focusing properties.

If polarizer P_2 moves with constant acceleration, starting from a dead stop, the angular displacement of the directions of polarizations of P_1 and P_2 is given by

$$\theta = \frac{1}{2} \beta t^2 \quad (3.9)$$

This relationship, in combination with Eqs. 3.1, 3.3, and 3.5 produces

$$I(x) = 1 + M \cos \left[\pi \left(\frac{dk}{dx} \right) x^2 \right] \quad (3.10)$$

In terms of the rotational variable, f , an alternative representation is possible. Thus

$$I(x) = 1 + M \cos \left[\frac{2\pi}{v^2} \left(\frac{df}{dt} \right) x^2 \right] \quad (3.11)$$

where df/dt is the rate of change of rotational speed, or revolutions per unit time per unit time. This is the measurable quantity, and the derivative dk/dx is the required relationship, because it contains the frequency variation. Equating the angular arguments of Eqs. 3.10 and 3.11 gives an equation for the instantaneous change of spatial frequency

$$\frac{dk}{dx} = \frac{2}{v^2} \left(\frac{df}{dt} \right) x \quad (3.12)$$

This derivative, which defines a packing rate, i.e., spatial frequency per unit distance, is plotted in Fig. 3-3 for values having practical significance. To typify its use, consider the following example.

Assume a requirement for a pattern which varies linearly in frequency from 250 to 0 cycles per millimeter. This pattern is to be displayed over a distance of 0.50 inch. The precise packing rate required is computed by

$$\frac{dk}{dx} = \frac{250 \text{ cycles per millimeter}}{0.50 \text{ inch}} = 500 \text{ cycles per millimeter per inch.}$$

If a running time of 1 minute is arbitrarily assumed, the film transport speed must be 1/2 inch per minute. From Fig. 3-3, it can be seen that the required rotational acceleration is 1,600 rpm. This can be verified by consulting Fig. 3-2, which shows 250 cycles per millimeter (the end point) to be produced at a rotational speed of 1,600 rpm and an image plane speed of 1/2 inch per minute.

If a higher value for the lower frequency (zero cycles per millimeter in the above case) were desired, it would be necessary to start the acceleration after the polarizer had reached some fixed velocity. This procedure requires an addition to the angular argument of the previous equations, since these had assumed starting from a dead stop. The analysis of a constant rotational speed coupled with an accelerated film, or image plane transport, produces similar results. This analysis will not be presented here.

There is an interesting application of this method which could be of use for ascertaining preliminary values of the modulation transfer characteristics of photographic emulsions. The technique hinges on maintaining (1) constant exposure and (2) constant input modulation over the frequency range utilized. The latter consideration has already been covered and constitutes no particular problem. The former is difficult but not impossible to maintain.

The two major factors which govern exposure loss are (1) the previously discussed decrease of exposure with spatial frequency, and (2) the reciprocity failure characteristics of the photographic material. Shaw² corrects for the first of these effects by passing a neutral density wedge by the slit (he uses accelerated motion of the image plane), programmed as a function of spatial frequency. In the system, the angular position of polarizer P_3 could just as easily achieve the same purpose. Fig. 3-4 shows the attenuation (in terms of effective neutral density) required to equalize exposures of a pattern containing spatial frequencies of 1 to 1,000 cycles per millimeter. The dashed curves are typical of the modifications required in the graph in order to account for reciprocity failure. If a particular system design were under evaluation, a plot of effective neutral density versus polarizer rotation would be the next logical step. Keyed into the rotational and translation motions, the plot data would be programmed (by mechanical means) into attenuation versus angular location, which in turn would be time variant.

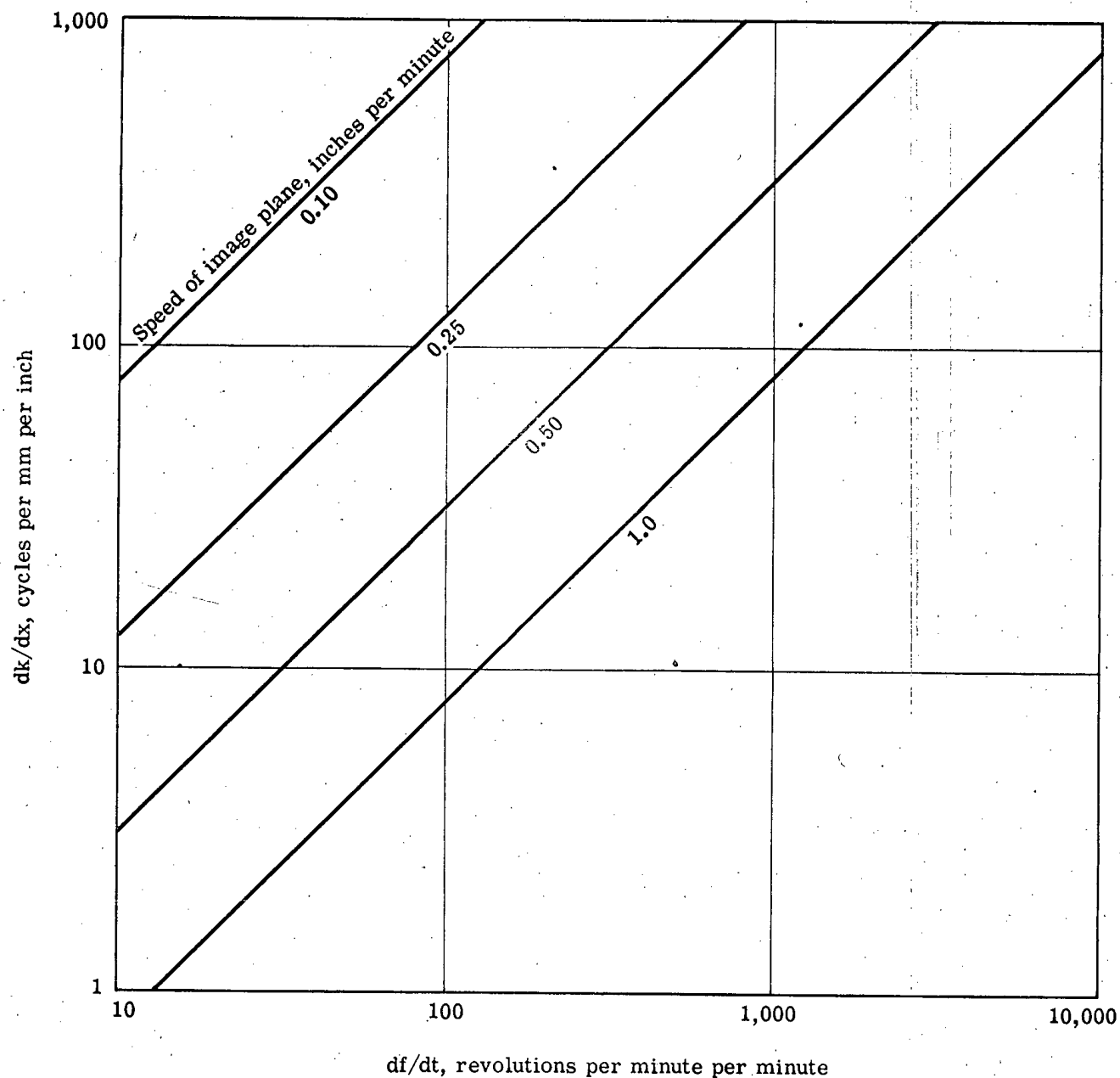


Fig. 3-3 — Parametric variation of spatial frequency packing rate with polarizer rotational acceleration and image (or film plane) translational velocity

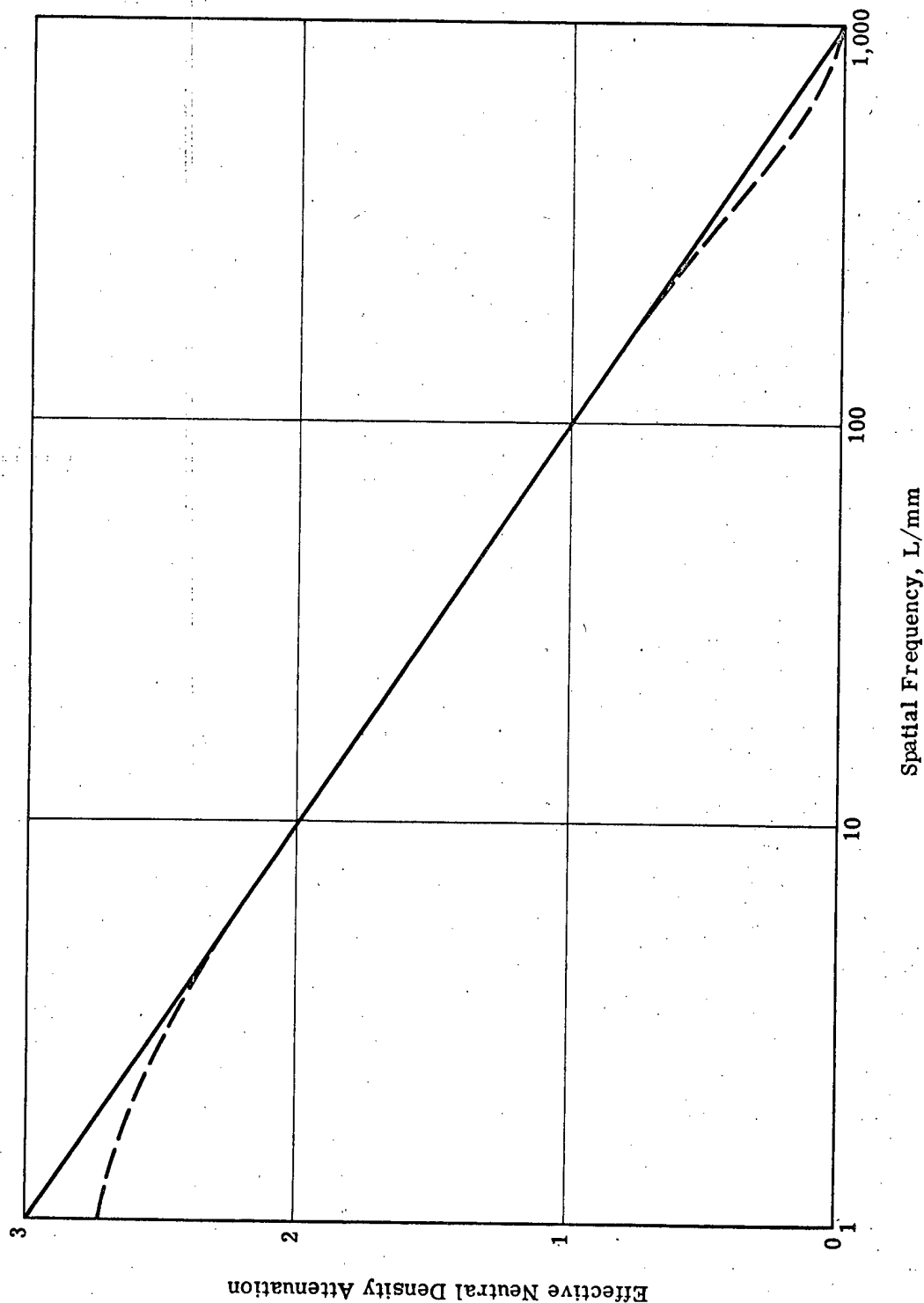


Fig. 3-4 — Modulation transfer function plot showing typical response modification necessary to equalize exposures from 1 to 1,000 cycles per millimeter

The density distribution on the film would then be of the form

$$D(x) = \gamma \log \left[1 + M(x) \cos (ax^2) \right] + \gamma \log A(x). \quad (3.13)$$

where $A(x)$ is a functional representation of the attenuating corrections required to account for exposure equalization for frequency and reciprocity effects. The sole purpose of this is to ensure a constant input light level so that the resultant pattern modification due to the modulation transfer characteristics of the photographic material can be more readily analyzed.

An alternative way to express (or acknowledge) the preceding relationship is

$$D(x) = D_0 + \gamma \log \left[1 + M(x) \cos (ax^2) \right] \quad (3.14)$$

It is explicitly understood that all exposures lie on the straight-line portion of the characteristic curve. Modulation, $M(x)$, is now a function of position on the image. The modulation diminishes as spatial frequency increases. Thus,

$$\lim_{x \rightarrow \infty} D(x) = D_0 \quad (3.15)$$

indicating that, when the input light level and modulation are held constant, the pattern maintains a constant, average density for all frequencies. If the correction for exposure loss is not made, $D(x)$ tends toward fog with increasing x , and the pattern's average density drops with frequency, i.e., there is no symmetry in the filmed image. This observation has been made at Itek in constructing such patterns by other means.

Now it can be seen that the loss in pattern contrast is entirely due to the modulation transfer characteristics of the photographic emulsion. Since the frequencies which exist are not discrete, but are more properly characterized by the term spatially instantaneous, it will not be possible to analyze them separately. However, it will be possible to extract $M(x)$ from a microdensitometer trace of the pattern. This is possible because the pattern envelope is closely related to the curve for the modulation transfer characteristics. It will be more useful to consider a trace in density, as there is symmetry about D_0 , and none about its corresponding T_0 .

Fig. 3-5 shows a sketch of such a pattern, using arbitrary scale values. This pattern only gives values of the envelope where tangency occurs, the remainder of the pattern having been interpolated. The analysis which follows gives an indication of what can be accomplished in this manner, although a much more rigorous treatment would be necessary to implement such a technique.

As stated previously, the function is assumed to be symmetrical about D_0 . The maxima and minima of this plot are given through standard methods, and after equating the first derivative to zero, the values of x which produce the maxima and minima are

$$x(n) = (n\pi/a)^{\frac{1}{2}} \quad (3.16)$$

where n is an integer, and the maxima and minima alternate as n takes on successive integral values. Since the pattern is symmetrical, only the upper envelope need be treated. The density maxima are given by

$$D(n) = \gamma \log \left[1 + M \left(\frac{n\pi}{a} \right) \right] + D_0, \quad n \text{ being even} \quad (3.17)$$

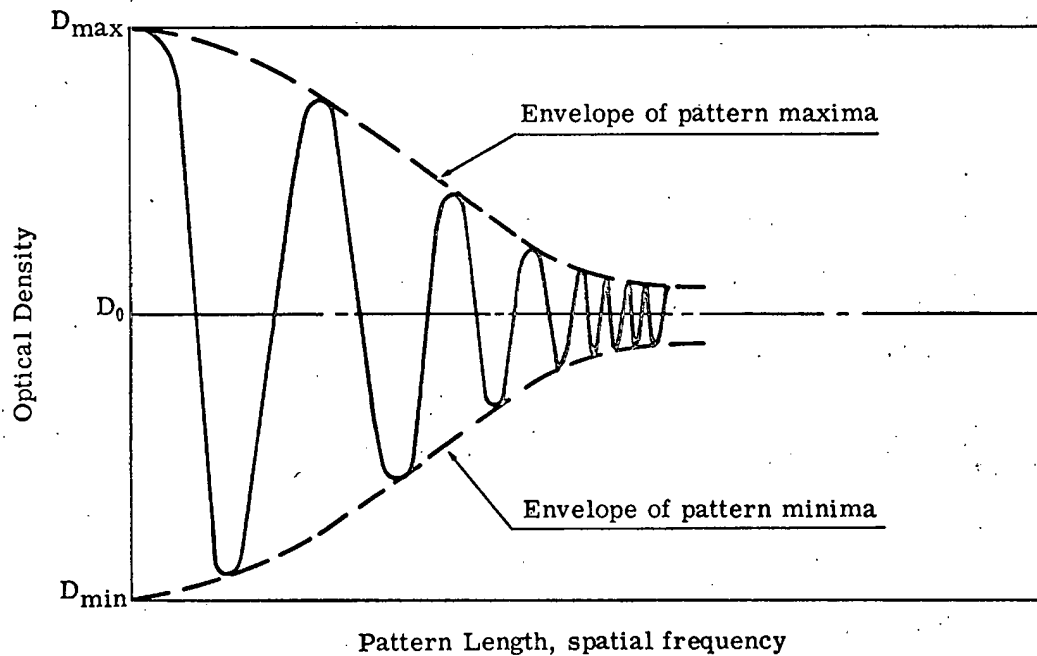


Fig. 3-5 — Sketch of multifrequency pattern produced when rotating polarizers are accelerated and exposure is corrected for frequency and reciprocity variations

The modulation transfer characteristic at the points defined by n results from the solution of Eq. (3.17), and

$$M \frac{n\pi}{a} = \exp(2.302) \{ [D(n) - D_0] / \gamma \} - 1 \quad (3.18)$$

By taking the values of the trace at the central maxima, points are provided on the modulation transfer curve, which is then drawn. A similar analysis leads to a formula for treating the trace minima. This formula can be used to increase the accuracy of the resulting plot. While the method as outlined is not of particularly high precision, it could serve to establish provisional data on a given material. Attention to mechanical, optical, photographic, and analytical details could improve the precision (as Shaw² has obviously done) and give an accurate measure of response, but it is not recommended that this latter technique be implemented.

3.3 REFERENCES

1. E. Ingelstam, E. Djurle, and B. Sjogren, Contrast-Transmission Functions Determined Experimentally for Asymmetrical Images and for the Combination of Lens and Photographic Emulsion, JOSA 46 (Sept. 1956).
2. R. Shaw, The Application of Fourier Techniques and Information Theory in the Assessment of Photographic Image Quality, PS&E 6 (Sept. - Oct. 1962).